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Collision efficiencies empirically determined from laboratory investigations of collisional growth of small raindrops in a laminar flow field

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Abstract

In laboratory experiments at the vertical wind tunnel of the University of Mainz the collisional growth of drops with radii between 70 and 170 μm in radius were observed while the collector drop freely floated in a cloud of droplets with radii ranging from 1 to 7 μm. Previously existing tables with collision efficiency values were interpolated and completed in such a way that drop growth rates calculated with these collision efficiencies match with observed growth rates. These new tables provide collision efficiency values for a wide range of drop sizes and radius ratios *p* including those ranges where efficiency values missed so far. This is of high importance for small *p*-ratios where the collision efficiency changes significantly. The empirically derived collision efficiencies can be used in cloud models. Growth rates observed during the laboratory experiments on drop collisions and those obtained by using model simulations of continuous growth using theoretically determined values of the collision efficiencies are in very good agreement.

Keywords: Laboratory experiments; Wind tunnel; Collision efficiency; Water drops

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1. Introduction

The collision efficiency of drops is the main factor controlling the drop collection rate and warm rain formation in liquid water clouds. It is defined as the ratio of the number of drops actually colliding with a larger falling drop (the collector drop) to the total number of drops lying within the geometrical volume of interaction and depends strongly on the sizes of the collision partners. The efficiency values lie, in general, between values close to zero (no collisions) and one; values higher than one may occur owing to wake capture of droplets in the rear eddies of the larger collision partner.

So far, experimental verification of theoretically calculated collision efficiencies is rather sparse. Drop collisions were studied experimentally by, e.g., Kinzer and Cobb (1958), Woods and Mason (1964), Beard and Pruppacher (1971), Jonas and Goldsmith (1972), Vohl et al. (1999). The lack of reliable data exists particularly for collisions between cloud droplets themselves and between small raindrops and cloud droplets. To the best of our knowledge, referenced tables of collision

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efficiencies do not contain data concerning collisions of 1 to 3 μm radius cloud droplets with raindrops (see, e.g., Hall, 1980; Beard and Ochs, 1984) corresponding with so-called *p*-ratios (the ratios of radii of colliding drops) as low as around 0.05. According to Pruppacher and Klett (1997) the collision efficiency increases very strongly with *p* for *p*-values less than 0.1 that means small variations of the *p*-ratio lead to significant changes in the collision efficiency. Therefore, the investigation of these size ranges is highly important. The knowledge of the efficiencies of these collisions is important for appropriate description of 1) comparable contribution of diffusion and collision growth of drops to drop spectrum evolution, and 2) evaluation of the rate of scavenging of micrometer-size droplets and aerosol particles from clouds by collisional processes.

In order to obtain data for the missing range of *p*-values, laboratory experiments were conducted on the continuous growth of small raindrops while the drops freely float within a cloud of droplets. These experiments were carried out in laminar flow using the vertical wind tunnel at the University of Mainz under controlled environmental conditions. From these experimental results collision efficiencies were determined empirically to fill the gaps mentioned above. In the end it was proven if the empirically determined collision efficiencies and the experimentally obtained drop growth rates verify collision efficiencies and growth rates numerically calculated by Pinsky et al. (2001).

2. Laboratory experiments

The experiments were carried out in the vertical wind tunnel at the University of Mainz which allows floating drops of micrometer to millimeter size range in a vertical air stream. For its entire range of air speeds the tunnel shows a turbulence level of less than 0.5% (Vohl et al., 1999). For details of the wind tunnel see Pruppacher (1988). During the present experiments a single collector drop was injected into the tunnel set at a speed matching the terminal velocity of the drop so that the drop was captured within the observation section of the tunnel in a free floating condition. The radius of this collector drop was between 70 and 170 μm. Upstream of the floating drop a cloud of droplets was produced by a battery of sprayers so that the larger collector drop grew by collision with the droplets. The droplet size spectrum was determined using a light scattering size spectrometer (Classical Scattering Aerosol Spectrometer Probe Electronics (CSASPE) from Particle Measuring Systems, Inc., Boulder, Colorado, USA). The droplet radii were between 1 and 7 μm.

The total liquid water content of the droplet cloud was determined from the actual temperature and the dew point of the tunnel air. To measure the dew point the tunnel air was sucked isokinetically into a heated inlet tube so that the droplets evaporated. The liquid water content of the droplet cloud was kept as stable as possible by switching the sprayers successively on as the tunnel air speed increased. Examples of typical size distributions of the small droplets and of the liquid water content of the cloud of small drops during the experiments can be found in Vohl et al. (1999) and in Vohl (2000).

The growth rate of the collector drop was determined from the change in terminal velocity while it was kept continuously suspended in the observation section during its growth. The air velocities in the wind tunnel were converted into equivalent drop radii using the relation given by Beard (1976). For details see Vohl et al. (1999) and Vohl (2000). Regarding the *p*-ratio of the drop sizes it was between 0.006 and 0.1, i.e. in a range where the collision efficiency changes significantly with the *p*-ratio.

3. Empirical determination of the collision efficiencies

During a wind tunnel collisional growth experiment a single levitated collector drop is allowed to grow in a cloud of small droplets streaming past the collector

Table 1
Collision efficiencies empirically determined from experimental results for collector drop radii between 100 and 600 μm and *p*-ratios ≤ 0.5

p	r_1 (μm)						
	600	501	398	316	200	158	100
0.005	0.6	0.24	0.1	0.01	0.001	0.0003	0.0001
0.01	0.85	0.66	0.51	0.23	0.04	0.001	0.001
0.025	0.98	0.91	0.88	0.82	0.61	0.42	0.04
0.05	1	0.97	0.95	0.94	0.87	0.78	0.48
0.075	1	1	0.97	0.96	0.93	0.89	0.72
0.1	1	1	1	0.98	0.95	0.93	0.81
0.125	1	1	1	1	0.96	0.95	0.87
0.15	1	1	1	1	0.97	0.96	0.91
0.175	1	1	1	1	0.98	0.96	0.93
0.2	1	1	1	1	0.99	0.97	0.94
0.225	1	1	1	1	1	0.98	0.95
0.25	1	1	1	1	1	1	0.96
0.275	1	1	1	1	1	1	0.965
0.3	1	1	1	1	1	1	0.97
0.325	1	1	1	1	1	1	0.98
0.35	1	1	1	1	1	1	0.99
0.375	1	1	1	1	1	1	1
0.4	1	1	1	1	1	1	1
0.425	1	1	1	1	1	1	1
0.45	1	1	1	1	1	1	1
0.475	1	1	1	1	1	1	1
0.5	1	1	1	1	1	1	1

drop. This can be best described by a continuous collisional drop growth process where the rate is given by the following equation (Pruppacher and Klett, 1997):

$$r_1^2 \frac{\mathrm{d}r_1}{\mathrm{d}t} = \frac{1}{3} \int K(r_1, r_2) r_3^2 n(r_2) \mathrm{d}r_2 \tag{1}$$

where

$$K(r_1, r_2) = E\pi (r_1 + r_2)^2 |V_{t,1} - V_{t,2}| \tag{2}$$

with K the collection kernel, r_1 and r_2 the radii of the collector drop and the droplet, respectively, $V_{t,1}$ and $V_{t,2}$ the corresponding terminal velocities. $E=E_{\mathrm{coll}}E_{\mathrm{coal}}$ is the collection efficiency with E_{coll} the collision efficiency, E_{coal} the coalescence efficiency. For the present estimations a coalescence efficiency of 1 was assumed, which means that all colliding drops flow together and form one larger drop. The coalescence efficiency can assume values in the range between 0 (drops do not flow together) and 1 (Pruppacher and Klett, 1997). According to Beard and Ochs (1984) the coalescence efficiencies for the colliding drops of the sizes investigated here are in the range of 0.9 and 1. Vohl et al. (1999) showed in sensitivity studies that the effect of various coalescence efficiency values was within the error bars of the experiments. As has been mentioned earlier, collision efficiencies between drops with values of the p-ratio $p=r_2/r_1$ (r_2 the radius of the smaller collision partner, r_1 the radius of the collector drop) as low as 0.05 are not available in literature. In order to determine these values from the present experiments an empirical approach was employed as follows.

Table 2
Collision efficiencies empirically determined from experimental results for collector drop radii between 10 and 79 μm and p-ratios ≤ 0.5

p	r_1 (μm)						
	79	64	50	40	30	20	10
0.005	–	–	–	–	–	–	–
0.01	–	–	–	–	–	–	–
0.025	0.002	0.0003	0.0002	–	–	–	0.001
0.05	0.23	0.03	0.003	0.0001	–	0.0001	0.0001
0.075	0.54	0.28	0.03	0.001	0.0001	0.001	0.005
0.1	0.68	0.45	0.2	0.01	0.001	0.007	0.01
0.125	0.76	0.59	0.28	0.07	0.015	0.011	0.011
0.15	0.83	0.69	0.39	0.16	0.031	0.015	0.013
0.175	0.86	0.74	0.49	0.27	0.064	0.02	0.015
0.2	0.89	0.77	0.58	0.36	0.12	0.026	0.017
0.225	0.91	0.8	0.65	0.45	0.18	0.033	0.019
0.25	0.92	0.83	0.7	0.51	0.23	0.04	0.022
0.275	0.93	0.86	0.715	0.56	0.28	0.048	0.024
0.3	0.94	0.88	0.73	0.6	0.32	0.058	0.027
0.325	0.945	0.89	0.735	0.63	0.36	0.068	0.03
0.35	0.95	0.9	0.74	0.66	0.39	0.079	0.032
0.375	0.96	0.91	0.745	0.68	0.42	0.091	0.035
0.4	0.97	0.92	0.75	0.7	0.45	0.1	0.038
0.425	0.97	0.92	0.76	0.72	0.47	0.115	0.04
0.45	0.97	0.92	0.77	0.73	0.49	0.13	0.043
0.475	0.97	0.93	0.78	0.74	0.51	0.145	0.45
0.5	0.97	0.93	0.79	0.74	0.53	0.155	0.046

Table 3
Collision efficiencies empirically determined from experimental results for collector drop radii between 10 and 600 μm and p-ratios > 0.5

p	r_1 (μm)							
	600, 501, 398, 316, 200, 158, 100	79	64	50	40	30	20	10
0.525	1	0.97	0.93	0.8	0.74	0.54	0.165	0.048
0.55	1	0.97	0.93	0.81	0.74	0.54	0.17	0.049
0.575	1	0.97	0.93	0.82	0.74	0.54	0.18	0.051
0.6	1	0.97	0.93	0.83	0.74	0.54	0.18	0.052
0.625	1	0.975	0.93	0.84	0.74	0.54	0.18	0.052
0.65	1	0.98	0.93	0.85	0.74	0.54	0.18	0.053
0.675	1	0.98	0.93	0.86	0.74	0.54	0.175	0.053
0.7	1	0.98	0.93	0.87	0.74	0.53	0.17	0.053
0.725	1	0.985	0.93	0.88	0.74	0.52	0.165	0.053
0.75	1	0.99	0.93	0.89	0.74	0.51	0.155	0.053
0.775	1	0.99	0.94	0.9	0.74	0.5	0.15	0.053
0.8	1	1	0.95	0.91	0.74	0.48	0.14	0.053
0.825	1	1.01	0.97	0.91	0.75	0.47	0.135	0.053
0.85	1	1.02	1	0.92	0.76	0.45	0.13	0.053
0.875	1	1.03	1.015	0.96	0.78	0.44	0.12	0.053
0.9	1	1.04	1.03	1.01	0.81	0.43	0.115	0.052
0.925	1	1.5	1.3	1.1	0.87	0.42	0.11	0.052
0.95	1	2.3	1.7	1.3	0.95	0.44	0.11	0.052
0.975	1	3	2.3	1.8	1.1	0.47	0.12	0.054
1.0	1	4	3	2.3	1.4	0.54	0.13	0.06

The Hall (1980) table contains collector drop sizes of 10 to 300 μm radius colliding with small drops with radius ratios from 0.05 to 1. It is based on model computations taking into account the findings of Jonas and Goldsmith (1972), Klett and Davis (1973), Lin and Lee (1975), and Schlamp et al. (1976). In the present study, some data in this table were adjusted. For collector drop radii smaller than and equal to 30 μm, the Jonas and Goldsmith (1972) data were omitted because they are based on Stokes flow with Cunningham slip correction factor leading to an underestimation of the collision efficiencies in this size range. Therefore, the new data for collector drop radii smaller than and equal to 30 μm include the findings of Klett and Davis (1973), Lin and Lee (1975), and Schlamp et al. (1976). They were obtained by using the non-statistical interpolation method kriging where unknown values are estimated by weighted medians of the known neighbor values. The value for collector drop radius of 70 μm and radius ratio 0.9 was replaced by the corresponding value given by Schlamp et al. (1976).

Afterwards, to extend the modified Hall table to larger drop sizes, it was combined with the findings of Beard and Ochs (1984) who investigated collector drop radii from 50 to 501 μm colliding with cloud droplets in the size range of 1.58 to 31.6 μm radius. In size ranges where no data are available in the original Hall table (1980), values from Beard and Ochs (1984) were included into the present table. Towards small values of *p*-ratio, the collision efficiency values of Beard and Ochs (1984) were extrapolated. These small *p*-ratios are highly important because the slope of the collision efficiency curves is very significant for small *p*-ratios. At small *p*-ratios where the Beard and Ochs (1984) table and the modified Hall table overlap, the values were interpolated in such a way that being used in Eq. (1) for continuous collisional drop growth they provided an excellent agreement between calculated growth rates and those observed in the wind tunnel experiments (for details see Vohl et al., 1999).

4. Theoretical calculation of the collision efficiencies

After the empirical determination of the collision efficiencies it was investigated how these values and the experimentally obtained drop growth rates compare with the collision efficiencies and growth rates numerically calculated by Pinsky et al. (2001). Pinsky et al. used the superposition method which means that each drop is assumed to move in a flow field induced by its counterpart moving alone. Neglecting electrical effects, a drop about to collide with another one experiences three main forces: gravity, buoyancy and drag force. The drag force can be represented as a sum of the drag force arising in calm air while the drop is falling with its terminal velocity and the force arising due to the influence of the velocity field induced by its counterpart. When the separation distance between the drops is large enough so that the hydrodynamic interaction is negligible each drop falls with its terminal fall velocity V_t induced by gravity. The terminal fall velocities are calculated following Beard (1976). To compute the velocity fields induced by moving drops the method of matching of two analytical solutions is utilized: the Stokes solution valid for small droplets ($Re<0.4$) and the Hamielec and Johnson (1962) solution valid for drops with Reynolds numbers up to 100 (corresponding to drop radii of about 300 μm). The equations of drop motion are solved using the fifth-order Runge–Kutta method. For more details see Pinsky et al. (2001).

5. Results

The collision efficiencies empirically determined from the experimental results are given in Tables 1–3 for different collector size ranges and *p*-ratios. In these tables values of the collision efficiency for a wide range of drop sizes are given including those ranges where reliable values missed so far. Tables 1–3 contain values based directly on the present experiments, i.e. for r_1

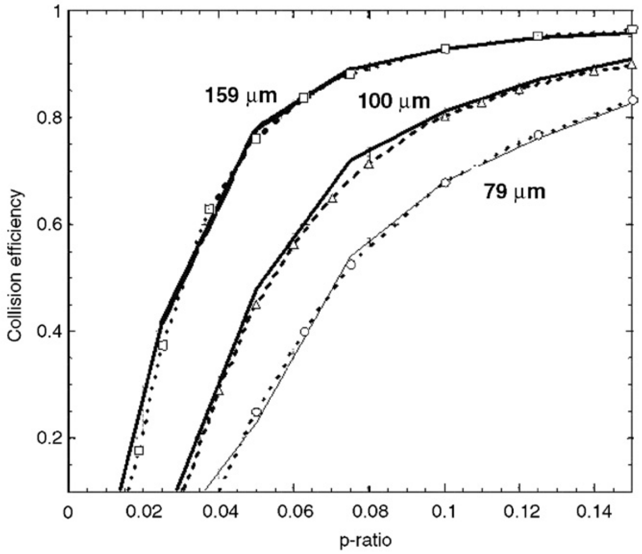

Fig. 1. Collision efficiencies determined empirically (solid lines) and those obtained using the numerical approach (dashed lines) as a function of the *p*-ratio for different radii of collector drops.

between 70 and 170 μm and *p*-ratios between 0.1 and 0.006, together with values originating from interpolations and extrapolations as described in Section 3.

Fig. 1 shows the collision efficiencies determined empirically from the laboratory experiments (solid lines) and those obtained using the numerical model of Pinsky et al. (2001) (dashed lines) for collector drop radii of 79 μm, 100 μm, and 159 μm and for small values of the *p*-ratio. One can see a very good agreement between the values calculated by the numerical model and those empirically determined from the laboratory experiments. A number of cloud microphysical models use the Hall (1980) table (e.g., Flossmann, 1997; Kerkweg et al., 2003; Wood, 2005; Simmel et al., 2005) to calculate drop collisions. To avoid implementing a numerical model as, e.g., the one of Pinsky et al. (2001), the new tables presented here can be used.

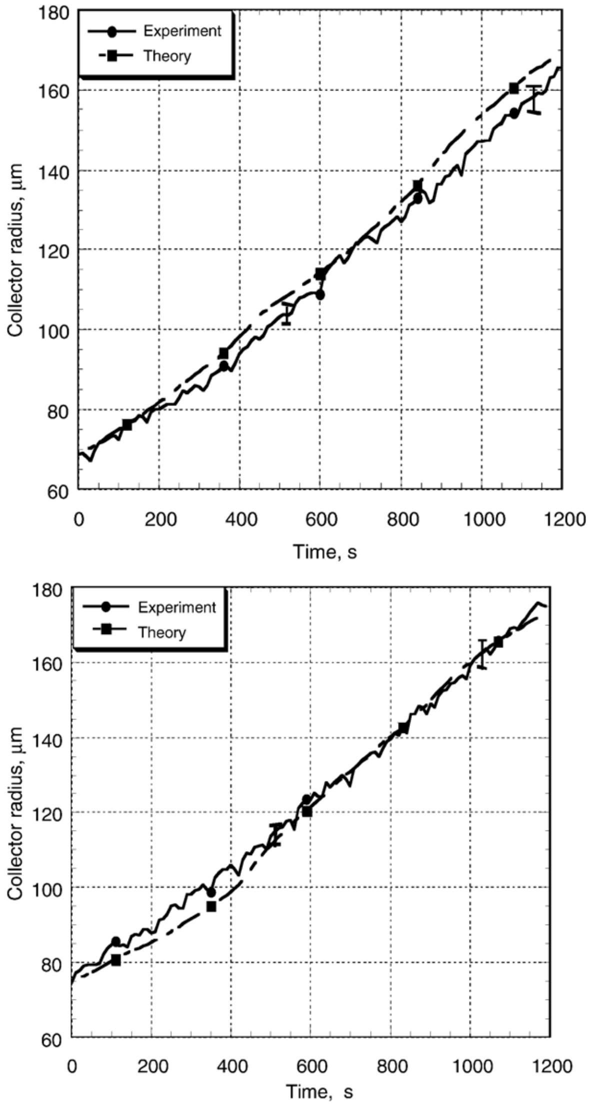

Fig. 2. Two examples of the increase of drop size vs. time as obtained from laboratory measurements and from calculations according to Pinsky et al. (2001). Initial collector drop sizes 70 μm (upper panel) and 75 μm (lower panel). The two error bars in each growth curve are representative for typical uncertainties in the experimental determination of the collector drop size.

Fig. 2 presents comparisons between a calculated collisional drop growth rate according to Pinsky et al. (2001) and the drop growth rate observed in the experiments. Two examples are shown for the increase of the collector drop size with time. One can see a good agreement between the measurements and the calculations. That means the present experiments verify the method of Pinsky et al. (2001) to calculate collision efficiencies. Assuming coalescence efficiency to be equal to unity, collision efficiency values may lead to an underestimation in the range of 0 to 10% of the collision efficiencies.

6. Conclusions

Two conclusions can be drawn from the present study:

a) Measurements of the growth rate of a single drop by collision with small droplets carried out in a wind tunnel provide a useful tool to verify theoretical collision efficiencies under the assumption that the coalescence efficiency is unity.
b) The agreement between collision efficiencies empirically determined from experimental results and numerically computed ones provides a high resolution table of the collision efficiencies for a wide range of drop sizes including those ranges where values (or at least reliable values) missed so far. This is of significant importance especially for small *p*-ratios. The collision efficiency values from Tables 1 to 3 can be used as an alternative to the Hall (1980) table for cloud model calculations.

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